CENTRAL COMPOSITE DESIGN

In statistics, a central composite design is an experimental design, useful in response surface methodology, for building a second order (quadratic) model for the response variable without needing to use a complete three-level factorial experiment.

This design was first introduced in a journal article called On the Experimental Attainment of Optimum Conditions by G. E. P. Box and K. B. Wilson in the year 1951. It was developed specifically for use in response surface exploration in order that the data collection phase be performed as completely, as cheaply, and as efficiently as possible.

Statistical approaches such as Response Surface Methodology can be employed to maximize the production of a special substance by optimization of operational factors. In contrast to conventional methods, the interaction among process variables can be determined by statistical techniques. For instance, in a study, a central composite design was employed to investigate the effect of critical parameters of organosolv pretreatment of rice straw including temperature, time, and ethanol concentration. The residual solid, lignin recovery, and hydrogen yield were selected as the response variables.

> A Box-Wilson Central Composite Design, commonly called 'a central composite design,' contains an imbedded factorial or fractional factorial design with center points that is augmented with a group of 'star points' that allow estimation of curvature.
>
> CCD enables estimation of the regression parameters to fit a second-degree polynomial regression model to a given response. A polynomial, as given by Equation (1), quantifies relationships among the measured response y and a number of experimental variables $X_1 \ldots X_k$, where k is the number of factors considered, β are regressors and ε is an error associated with the model:

$$\begin{aligned} y = \beta_0 &+ \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k \\ &+ \beta_{11} X_1^2 + \beta_{22} X_2^2 + \ldots + \beta_{kk} X_k^2 \\ &+ \beta_{12} X_1 X_2 + \ldots + \beta_{k-1,k} X_{k-1} X_k + \varepsilon \end{aligned}$$

CCD requires three types of trials, i.e., 2^k factorial trials, 2k axial trials and n_c center point trials, where k is number of factors studied in the experiment. As an example, this is illustrated in Figure, where each point defines a unique set of values of experimental trials for the three factors (k = 3) tested in an experiment.

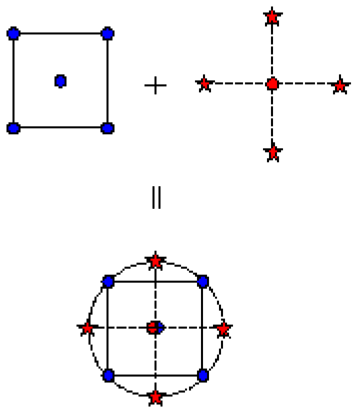

Fig4.4: Generation of central composite design for two factors

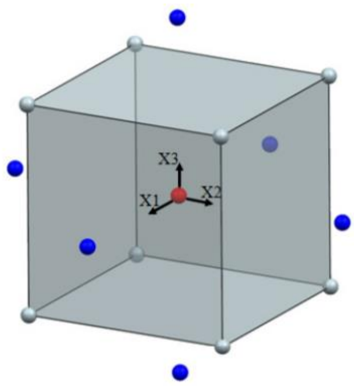

Fig4.5: Visualization of original type rotatable CCD for three factors: X1, X2 and X3.

Values at the center point (red point with coordinates 0, 0, 0) that is located in the center of the cube are used to detect curvature in the response, i.e., they contribute to the estimation of the coefficients of quadratic terms. Axial points (six blue points located at a distance α from the center point) are also used to estimate the coefficients of quadratic terms, while factorial points (eight grey points located at corners of the cube with a side length equal to 2) are used mainly to estimate the coefficients of linear terms and two-way interactions. For testing four or more factors in an experiment, Figure should be extended to the fourth or more dimensions.

In CCD, factors are tested at a minimum of three levels: minimum, middle and maximum, equivalent to levels −1, 0 and 1, which are called coded units. If X_{min} and X_{max} are respectively minimum and maximum absolute, i.e., uncoded, values of a factor, the absolute values X corresponding the respective coded values can be obtained by a simple linear transformation of the original measurement scale, namely:

$$X = b \cdot coded\ value + a$$

where:

$$a = \frac{X_{max} + X_{min}}{2}, b = \frac{X_{max} - X_{min}}{2}$$

There are three types of central composite design:

1. **Circumscribed CCD:** is the original type of CCD, where axial points are located at distance α from the center point.
2. **Face-centered CCD:** In face-centered CCD that is presented in Figure 4, axial points are located at a distance 1 from the center point, i.e., at the face of the design cube, if the design involves three experimental factors.
3. **Inscribed CCD:** In turn, inscribed CCD is characterized by that axial points are located at factor levels −1 and 1, while factorial points are brought into the interior of the design space and located at distance 1/α from the center point.

   Geometrically, inscribed CCD reminds of circumscribed CCD. This is because factorial points are set at a distance from the center point so that these distances between the factorial points, the axial points and the center point are in the same proportion as in circumscribed CCD.

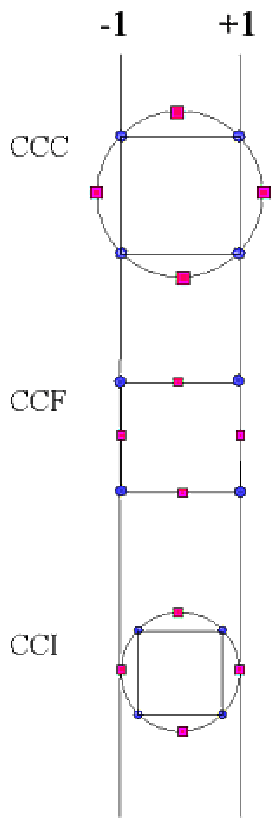

Fig4.6: Comparison of the types of CCD

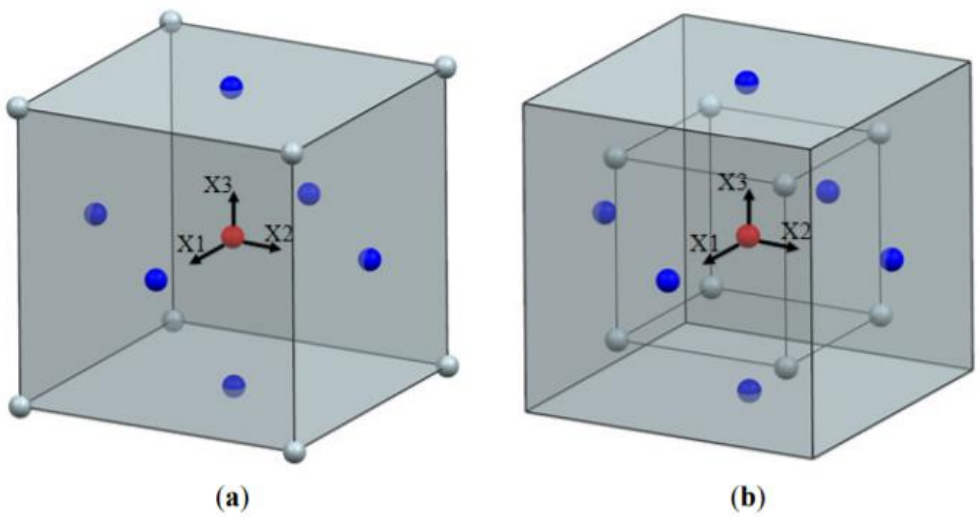

Fig4.7:Visualization of (a) face-centered CCD (b) and inscribed CCD for three factors: X1, X2 and X3.

4.2.2.2 ADVANTAGES OF CCD

The advantages of Central Composite Design are:

- In Central Composite Design the minimum numbers of factors it can accommodate is

2 whereas in Box-Behnken Design the minimum number of factors (continuous or numerical factors) it can accommodate is 3.

- Central Composite design is a response surface design which apart from the 3 level factors has axial or star point. The axial or star point usually denoted as (α) increases the number of levels to 5 levels thereby giving the experimental design flexibility.
- Its advantages over Box-Behnken is that it allows the experimental designer to know what effect the factors had on response is the experimental designer goes beyond or below the chosen levels of factors.

4.2.2.3 DISADVANTAGES OF CCD

The disadvantages of Central Composite Design are:

- The central-composite design requires a rather rigid pattern of data collection points which do not always fit the needs of human factors engineering studies. Five levels of each factors are required. They must be spaced symmetrically about the center at particular locations on a scale, which changes as the number of factors in the study change.
- Under similar conditions, often the number of tests is greater than Box-Behnken.

4.2.2.4. APPLICATIONS

- One important area where computer simulation is used a lot is engineering. Engineers are confronted with the task of designing products and processes. Since physical experimentation is often expensive and difficult, computer models are frequently used for simulating physical characteristics.
- In the field of research and development sector.
- Central composite design is used worldwide in computer modelling related to performing safety assessments.
- Optimization of operational factors.
- Chemical and engineering research.
- Applied human factors engineering research.
- Neural networks.
- It is also used in computer modelling of reliability analyses.